

On the enhancement of nuclear reaction rates in high-temperature plasma

M. Nakamura^{a,*}, V.T. Voronchev^b, Y. Nakao^a

^a*Department of Applied Quantum Physics and Nuclear Engineering, Kyushu University, Hakozaki, Fukuoka 812-0064, Japan*

^b*Institute of Nuclear Physics, Moscow State University, Moscow 119992, Russia*

Abstract

We argue that the Maxwellian approximation can essentially underestimate the rates of some nuclear reactions in hot plasma under conditions very close to thermal equilibrium. This phenomenon is demonstrated explicitly on the example of reactions in self-sustained DT fusion plasma with admixture of light elements $X = \text{Li, Be, C}$. A kinetic analysis shows that the reactivity enhancement results from non-Maxwellian knock-on perturbations of ion distributions caused by close collisions with energetic fusion products. It is found that although the fraction of the knock-on ions is small, these particles appreciably affect the $\text{D}+\text{X}$ and $\text{T}+\text{X}$ reaction rates. The phenomenon discussed is likely to have general nature and can play role in other laboratory and probably astrophysical plasma processes.

Key words: hot plasma, nuclear reaction rate, ion distribution function

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1 Introduction

The concept of nuclear reaction rate is broadly used in high-temperature plasma research. This rate determines reaction yield and ultimately specific nuclear power released in plasma. The yield of a reaction between plasma species 1 and 2 is given by

$$Y(1+2) = \alpha n_1 n_2 \langle \sigma v \rangle_{12}, \quad (1)$$

* Corresponding author.

Email address: nakamura@nucl.kyushu-u.ac.jp (M. Nakamura).

where $\alpha = 1/2$ or 1 for identical or different colliding nuclei, respectively, n_1 and n_2 are species densities. The key quantity in (1) is the reaction rate parameter $\langle\sigma v\rangle_{12}$ defined as the six-dimensional integral in velocity space

$$\langle\sigma v\rangle_{12} = \int f_1^u(\mathbf{v}_1) f_2^u(\mathbf{v}_2) \sigma(|\mathbf{v}_1 - \mathbf{v}_2|) |\mathbf{v}_1 - \mathbf{v}_2| d\mathbf{v}_1 d\mathbf{v}_2. \quad (2)$$

Here σ is the 1+2 reaction cross section, \mathbf{v}_1 and \mathbf{v}_2 are particle velocities in the laboratory frame, f_1^u and f_2^u are unit-normalized particle velocity distribution functions. In a number of cases the integral (2) can be simplified. For example, in Maxwellian plasma $\langle\sigma v\rangle_{12}$ takes the well-known form

$$\langle\sigma v\rangle_{12} = \left(\frac{8}{\pi\mu}\right)^{1/2} \frac{1}{T^{3/2}} \int_0^\infty E \sigma(E) \exp\left(-\frac{E}{T}\right) dE, \quad (3)$$

where μ is the reduced mass of the colliding particles, E is their kinetic energy in the center-of-mass frame, T is the plasma ion temperature. The Maxwellian approximation is a conventional tool to study plasma under conditions close to thermal equilibrium. At the same time, however, ion distribution functions in high-temperature plasma strictly speaking are not purely Maxwellian. A reason of non-Maxwellian deviation lies in exothermic nuclear reactions proceeding in the plasma. These reactions generate energetic projectiles which during slowing-down affect the formation of ion distributions. Charged particles slow down in the plasma mainly via peripheral (small-angle) Coulomb scattering by thermal ions and electrons. This mechanism does not change the equilibrium form of ion distribution. However, since the energy of reaction products can reach several MeV, close (large-angle) collisions between them and thermal ions can also take place in the plasma. The probability of close collisions is determined by amplitudes of Coulomb and nuclear scattering, and their interference term. Although such processes occur at rare opportunity, they can transfer in a single event a large amount of energy and produce fast knock-on ions. These ions increase the population of high-energy tails of respective distributions, so that some deviation from Maxwellian functions appears. Apart from energetic charged particles, reaction-produced neutrons can also contribute to the knock-on perturbation mechanism if the plasma is sufficiently dense.

Since the Maxwellian approximation has widely been used in laboratory and astrophysical plasma studies, a natural question arises whether the knock-on perturbation of ion distribution could in some cases appreciably change reaction rates and affect power balance in hot plasma systems. It has been recognized that for conventional DT and DD fusion plasmas the answer is negative. The combination of three factors – sizable reaction probabilities at thermal energies where the majority of ion population is concentrated, small fractions of knock-on deuterons and tritons, and moderate energy dependences of the fusion cross sections in the energy range associated with these fast ions –

makes the above processes poorly sensitive to slight modifications of D and T distribution tails. This especially concerns the resonant D+T reaction whose cross section has a broad maximum at deep sub-barrier energies. However, in systems composed of nuclei with $Z > 1$ the situation has still been intriguing. In such a system strong Coulomb repulsion suppresses transmission probability through the potential barrier between interacting nuclei and, in the absence of pronounced low-energy resonances, the behavior of reaction cross section becomes steep at least at sub-barrier (sub-MeV) energy range. This suggests that the respective reaction can be sensitive to the form of ion distribution tail, so that the suprathermal reaction channel induced by knock-on ions may become appreciable. The purpose of this letter is to investigate the possible enhancement of nuclear reaction rates due to knock-on perturbations of ion distributions.

2 Semi-qualitative consideration

One can reproduce such situation in a simple two-temperature model. Let us describe ion distributions in a plasma as superposition of two functions $f + f'$. The first one is Maxwellian; it represents the behavior of bulk ions with density n and temperature T . The second function is introduced to model the ensemble of knock-on ions with density $n' < n$. We assume that f' also is Maxwellian with some temperature $T' > T$. Then the total reaction yield (1) can be presented as

$$Y(1+2) = Y_{bulk} \times (1 + \lambda), \quad (4)$$

where $Y_{bulk} = \alpha n_1 n_2 R(T_1, T_2)$ is the thermal yield provided by bulk particles, while λ gives the suprathermal correction caused by bulk-fast and fast-fast ion interactions

$$\lambda = \frac{n'_1 R(T'_1, T_2)}{n_1 R(T_1, T_2)} \frac{1}{\alpha} + \frac{n'_2 R(T_1, T'_2)}{n_2 R(T_1, T_2)} \frac{1}{\alpha} + \frac{n'_1 n'_2 R(T'_1, T'_2)}{n_1 n_2 R(T_1, T_2)}. \quad (5)$$

Here R denotes $\langle \sigma v \rangle$ for the 1+2 reaction between Maxwellian species with different temperatures. Substituting Maxwellian distributions with temperatures θ_1 and θ_2 for f_1^u and f_2^u in (2), we find that the two-temperature rate parameter $R(\theta_1, \theta_2)$ can be reduced to the reactivity $R(T_{eff})$ for Maxwellian plasma (3) with some effective temperature T_{eff} :

$$R(\theta_1, \theta_2) = R(T_{eff}), \quad T_{eff} = \frac{m_2 \theta_1 + m_1 \theta_2}{m_1 + m_2}. \quad (6)$$

This allows one to easily estimate λ . Let us consider, as an example, different D+X systems: symmetric (X = D), nearly symmetric and resonant (X = T), asymmetric and involving light nuclei (X = Li, Be). We assume that only the D distribution is distorted, while the other particles are Maxwellian. It seems

reasonable to set bulk temperatures of D and X nearly equal, $T_D \simeq T_X = T$, and neglect the contribution of the fast-fast ion interaction term in (5). Under these conditions (5) takes the form

$$\lambda \simeq \frac{n'_D}{n_D} \frac{R(T'_D, T)}{R(T)} \frac{\beta}{\alpha}, \quad (7)$$

where β equals 2 ($X=D$) or 1 ($X \neq D$). Choosing $T = 10$ keV typical of fusion plasma level and varying the unknown temperature T'_D in the 50–200 keV wide range, we find that the ratio $R(T'_D, T)/R(T)$ changes approximately within 8–50 (D+D), 5×10^2 – 5×10^3 (D+ ^6Li), 10^3 – 5×10^4 (D+ ^7Li), 6×10^3 – 7×10^5 (D+ ^7Be), 8×10^3 – 6×10^5 (D+ ^9Be). For the D+T reaction this ratio changes from 6.29 to 6.35, *i.e.* proves to be nearly constant. Such invariant-like behavior with respect to T'_D results from resonant nature of the D+T reaction, due to which its reactivity rapidly increases at low temperature, peaks around 60 keV and then becomes essentially insensitive to plasma temperature. Thus, in the D+Li and D+Be systems a very small fraction of knock-on deuterons $n'_D/n_D < 0.1\%$ makes the contribution of thermal and suprathermal reaction components comparable, while for the D+T and D+D reactions the effect is rather imperceptible. One should keep in mind, however, that these results give approximate picture because the two-temperature model does not reproduce true form of particle distributions. Indeed, the knock-on ions are not Maxwellian; at least their distribution should be truncated at some critical energy determined by kinematics for particle collision. Nevertheless, the above estimations indicate that the problem really stands and it is worth studying rigorously.

3 Plasma kinetic analysis

In the present work we employ an appropriate plasma kinetic model to study the phenomenon on the example of various reactions in self-sustained DT fusion plasma with admixture of light elements. The concentration of these elements is assumed to be sufficiently low to neglect their role when analyzing the behavior of main plasma species – fuel ions and 3.5-MeV α -particles born in DT fusions. It was shown (see, for example, [1,2,3,4]) that these energetic α -particles are responsible for non-Maxwellian perturbations of fuel ion distributions due to α -D and α -T close collisions. We describe the behavior of plasma species a (deuterons, tritons, α -particles) with isotropic velocity distributions in terms of a Boltzmann-Fokker-Planck (BFP) equation. The BFP equation at steady-state without external heating can be written in the following form [4,5]:

$$\frac{\partial f_a}{\partial t} = \left(\frac{\partial f_a}{\partial t} \right)_{Coul.} + \left(\frac{\partial f_a}{\partial t} \right)_{NES} + \left(\frac{\partial f_a}{\partial t} \right)_{cond.} - L_a + S_a = 0, \quad (8)$$

where f_a is the density-normalized distribution function of species a . The plasma is assumed to satisfy the quasi-neutrality condition: $n_e = n_d + n_t + 2n_\alpha$. The first operator in the right hand of (8) represents the effect of small-angle a -ion and a -electron Coulomb scattering

$$\left(\frac{\partial f_a}{\partial t}\right)_{Coul.} = \frac{1}{v^2} \frac{\partial}{\partial v} \left(A_a f_a + B_a \frac{\partial f_a}{\partial v} \right), \quad (9)$$

where the functions A_a and B_a are given in [6,7,8]. The second operator is a Boltzmann collision integral describing the effect of close a - b collision ($b = d, t, \alpha$)

$$\begin{aligned} \left(\frac{\partial f_a}{\partial t}\right)_{NES} &= \sum_b \frac{2\pi}{v^2} \int_0^\infty v' f_a(v') \int_0^\infty v'_b f_b(v'_b) P(v' \rightarrow v|v_b) \\ &\times \left(\int_{|v'-v'_b|}^{v'+v'_b} v_r'^2 \sigma_{NES}(v_r') dv_r' \right) dv' dv'_b - \sum_b \frac{2\pi}{v} f_a(v) \int_0^\infty v_b f_b(v_b) \\ &\times \left(\int_{|v-v_b|}^{v+v_b} v_r^2 \sigma_{NES}(v_r) dv_r \right) dv_b. \end{aligned} \quad (10)$$

Here $v'_r = |v' - v'_b|$, $v_r = |v - v_b|$, $P(v' \rightarrow v|v_b)$ gives the probability distribution function for the speed v of a scattered particle, and σ_{NES} is the collision cross section quoted from [9]. The third term

$$\left(\frac{\partial f_a}{\partial t}\right)_{cond.} = \frac{1}{v^2} \frac{\partial}{\partial v} \left(\frac{v^3 f_a}{2\tau_C^{(a)}(v)} \right) \quad (11)$$

gives the diffusion in velocity space due to thermal conduction with the typical time $\tau_C^{(a)}$. Finally, L_a and S_a are particle loss and source terms, respectively, taking different forms for every ion species. Plasma electrons are considered to be Maxwellian at some temperature T_e incorporated in our model by using a global power-balance (GPB) equation. This equation determines the relation between plasma density and temperature, and has the simple form

$$P_{heat}(n_i, T_i) - P_{brem}(n_i, n_e, T_e) - P_{c.p.}(n_i, n_e, T_i, T_e) = 0. \quad (12)$$

Here P_{heat} is the plasma heating rate by α -particles, P_{brem} is the rate of bremsstrahlung energy loss, $P_{c.p.}$ gives the energy loss due to thermal conduction and particle leak. At chosen T_e , being an input parameter in our model, plasma density is estimated from (12) assuming $n_e \simeq n_d + n_t$ and $T_e \simeq T_i$. The detailed description of the kinetic model and explicit expressions for all terms in the BFP and GPB equations one can find in [4].

Figure 1 shows the particle distributions in energy space calculated under conditions close to ITER-like plasma. The non-Maxwellian perturbations of the deuteron and triton functions caused by α -D and α -T close collisions are

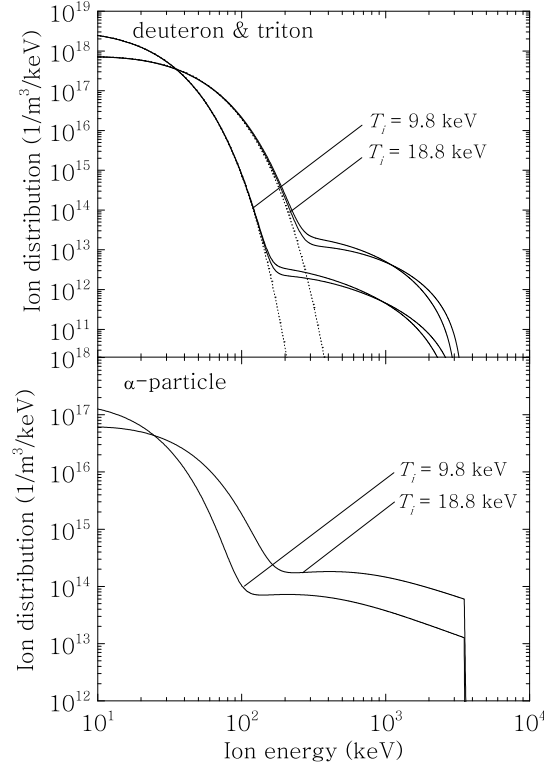


Fig. 1. The energy distribution functions of fuel ions and α -particles calculated under two plasma conditions: (i) $T_e = 10$ keV, $T_i = 9.8$ keV, $n_e = 1.2 \times 10^{20} \text{ m}^{-3}$, $n_d = n_t = 5.8 \times 10^{19} \text{ m}^{-3}$, and (ii) $T_e = 20$ keV, $T_i = 18.8$ keV, $n_e = 6.1 \times 10^{19} \text{ m}^{-3}$, $n_d = n_t = 2.8 \times 10^{19} \text{ m}^{-3}$. The respective Maxwellian distributions are shown by the dotted curves.

clearly marked at energies above a few hundred of keV.¹ The distribution of α -particles reflects well slowing down history of these fusion products. This distribution reveals moderate energy dependence in the 0.1–3.5 MeV deceleration range, while at thermal energies it is described by Maxwellian-like form. The plasma ion temperature T_i in Fig. 1 is evaluated as

$$T_i = (n_d^{bulk} T_d + n_t^{bulk} T_t) / (n_d^{bulk} + n_t^{bulk}), \quad (13)$$

where the temperatures of deuterons T_d and tritons T_t are obtained by fitting the bulk components of the D and T distributions to proper Maxwellian functions. For conditions considered in the work the difference between T_d and T_t does not exceed 4 %. Although it is rather small, we employ the general definition of T_i Eq. (13) instead of its reduction $T_d = T_t = T_i$ to estimate this basic plasma parameter as accurately as possible. An informative parameter is the

¹ We note that knock-on deuterons were already observed in DT fusion experiments at JET [10].

Table 1

The list of nuclear reactions in the DT/X plasma and the enhancement of their rate parameters caused by the knock-on deuterons and tritons

system	reaction	Q -value (MeV)	E_γ^a (MeV)	$\langle\sigma v\rangle/\langle\sigma v\rangle_{\text{Mxw}}$ $T_i = 10\text{--}40$ keV
D+Li	${}^6\text{Li}(d, n_1){}^7\text{Be}^*$	2.95	0.429	2.2–1.5
	${}^6\text{Li}(d, p_1){}^7\text{Li}^*$	4.55	0.478	2.5–1.5
	${}^6\text{Li}(d, pt)\alpha$	2.56		2.9–1.6
	${}^6\text{Li}(d, \alpha)\alpha$	22.37		1.5–1.2
T+Li	${}^6\text{Li}(t, d_1){}^7\text{Li}^*$	0.51	0.478	7.8–1.7
	${}^6\text{Li}(t, p_1){}^8\text{Li}^*$	- 0.18	0.981	$10^8\text{--}20$
D+Be	${}^9\text{Be}(d, \gamma){}^{11}\text{B}$	15.81		50–3
D+C	${}^{12}\text{C}(d, p_1){}^{13}\text{C}^*$	- 0.37	3.089	$10^{18}\text{--}10^4$
D+T	$\text{D}(t, n)\alpha$	17.59		≤ 1.01
D+D	$\text{D}(d, n){}^3\text{He}$	3.27		≤ 1.07

^a Energies of γ rays emitted by the excited daughter nuclei

fraction of the knock-on ions n'/n . It is estimated to be at the level of 0.03 % indicating that the plasma conditions are very close to thermal equilibrium.

Now we can examine the influence of the knock-on ions on reaction rates in the DT/X plasma. In the present study the admixture ions X are chosen to be Li, Be, C. These light elements are often considered as low- Z impurity in magnetic confinement fusion devices. For example, Li was already used in operation of TFTR [11], ASDEX and TEXTOR tokamaks, W-7 AS stellarator [12,13], and has been proposed as a diagnostic admixture for fusion machines of next generation [14,15,16]. Be and C have been vigorously used for plasma diagnostics in JET [17,18]. In order to make the study most informative we examine the variety of reactions having different mechanisms. They are listed in Table 1 including some processes proposed for plasma γ -ray spectroscopy, the energy-producing $\text{Li}(d, \alpha)$ and tritium-breeding $\text{Li}(d, pt)$ reactions, the conventional D+T and D+D fusion processes. The respective reaction cross sections are plotted in Fig. 2.

To calculate $\langle\sigma v\rangle$ for the distorted isotropic distributions of D and T, we reduce the general expression for reactivity (2) to the form

$$\langle\sigma v\rangle_{12} = \frac{8\pi^2}{n_1 n_2} \int_0^\infty v_1 f_1(v_1) \int_0^\infty v_2 f_2(v_2) \int_{|v_1-v_2|}^{v_1+v_2} v^2 \sigma(v) dv dv_1 dv_2, \quad (14)$$

where v is the relative speed $|\mathbf{v}_1 - \mathbf{v}_2|$. Assuming the comparatively heavy particles X to be Maxwellian, the reaction rate parameters have been com-

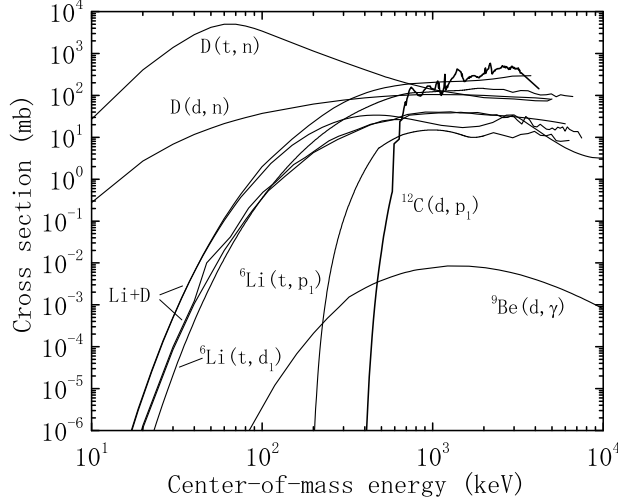


Fig. 2. The cross sections of reactions listed in Table 1. The D+Li curves are not resolved well at sub-barrier energies.

puted at ion temperature $T_i = 10\text{--}40$ keV. The results are plotted in Figs. 3 and 4, and also displayed in Table 1. We see that although the fraction of the knock-on ions is only 0.03%, these particles appreciably affect the D+X and T+X reactivity. It is underestimated if Maxwellian DT plasma is assumed, and for some reactions the discrepancy between the two approaches becomes crucial. The ratio $\delta = \langle \sigma v \rangle / \langle \sigma v \rangle_{\text{Mxw}}$ monotonically increases with decreasing T_i down to the plasma ignition point, and for the exothermic reactions presented in Fig. 3 it changes approximately within 1.5–2.5 (D+Li), 2–8 (T+Li) and 3–50 (D+Be). Special attention is worth being paid to the endothermic T+Li and D+C reactions displayed in Fig. 4. Here δ turns out to be several orders of magnitude or more, that reflects threshold nature of these processes. Both of them are forbidden at energies below thresholds, so only sufficiently fast ions contribute to the reactions. At the same time, Fig. 1 shows that the amount of these particles is essentially underestimated in Maxwellian plasma. The record enhancement is marked for D+C; the high threshold and strong Coulomb suppression of the thermal channel make the role of the knock-on deuterons extremely important here. Thus, the both reactions proceed via suprathermal channels and are solely governed by knock-on ions. This may have interesting applications in fusion technology. For example, 0.981-MeV photons emitted in ${}^6\text{Li}(t, p_1)$ might be applicable to energetic triton and α -particle diagnostics [4].

In agreement with the comments in Section 1 we find that the knock-on ions do not significantly affect the D+T and D+D reactions. Table 1 shows that the enhancement factor δ does not exceed 1% (D+T) and 7% (D+D).

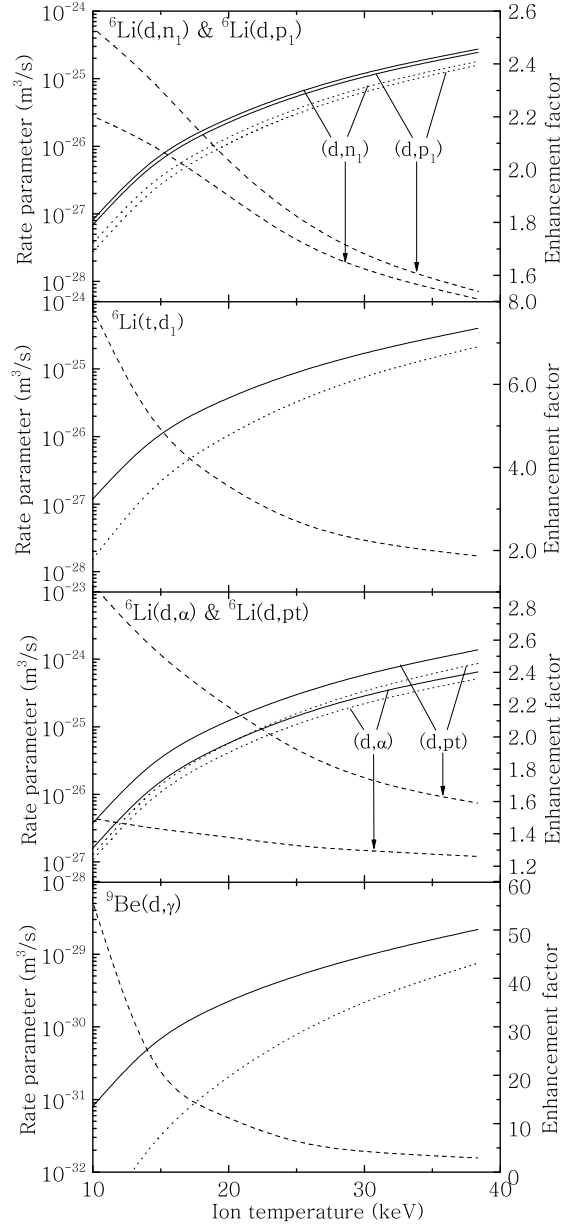


Fig. 3. The rate parameters for exothermic reactions from Table 1. The computed and Maxwellian data are shown by the solid and dotted curves, respectively. The dashed curves give the factor $\delta = \langle \sigma v \rangle / \langle \sigma v \rangle_{\text{Mxw}}$.

4 Conclusion

We have presented arguments that the Maxwellian approximation can essentially underestimate reaction rates in nearly thermal-equilibrium plasma, and explicitly demonstrated this phenomenon for some reactions in the DT/X plasma. The enhancement of reactivity results from the knock-on perturba-

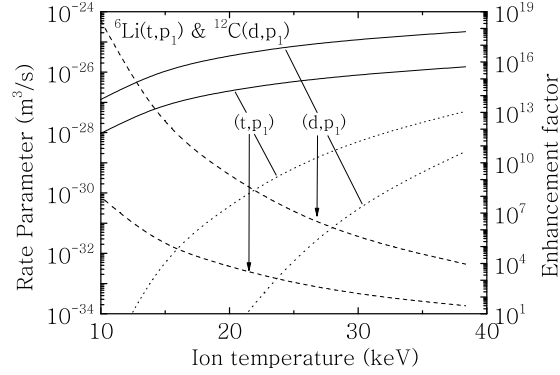


Fig. 4. The rate parameters for endothermic reactions from Table 1.

tions of ion distributions caused by close collisions with energetic fusion products. The numerical analysis carried out in the work is consistent with the semi-qualitative consideration, but the level of reactivity enhancement marked has turned out to be surprisingly high.

The phenomenon is likely to have general nature and can play role in other plasma systems. This especially concerns threshold nuclear processes – our study indicates that evaluation of their rates within the Maxwellian approximation can involve dramatic errors and prove to be fully useless. It seems possible that knock-on ions can affect power balance and even reduce ignition temperature for some exotic fuels, in which conditions favorable for the non-Maxwellian pumping of ion distributions can be realized. The aneutronic ${}^3\text{He}$ plasma would be an interesting object for such a study. Indeed, the ${}^3\text{He}+{}^3\text{He}$ reaction has large $Q = 12.9$ MeV, generates fast charged particles and exhibits an appropriate cross section behavior. The reaction cross section rapidly and monotonically rises with increasing energy up to the MeV region.

Apart from laboratory plasmas, the phenomenon discussed may also appear in some astrophysical processes. Primordial plasma is of particular importance here. Standard big-bang nucleosynthesis (BBN) relies on nuclear reaction network involving Maxwellian reactivity, and accuracy of nuclear inputs has been under attention [19,20]. If the mechanism of non-Maxwellian deviation would come into play under BBN specific conditions, it could update input reactivities and thereby offer new insight into synthesis of light elements in the early universe. Analysis of this scenario requires coupled cosmological and plasma kinetic calculations which were beyond the scope of our work. However, the demonstration that a very small, almost negligible fraction of knock-on particles ($\sim 10^{-4}$) in hot plasma can significantly change the rates of some reactions gives impetus to such study.

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